Creating Elementary Mathematics Lessons

with Children's Rich Imaginations

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Abstract

According to the report of TIMSS (1999), Japanese children's performance in mathematics is relatively high, but their attitudes towards mathematics are extremely negative; in fact they are at the lowest in the world. This is a serious problem for mathematics education in Japan. This article discusses ways to improve the current mathematics education system, presenting concrete examples.

To improve mathematics education, the teacher has to consider the importance of children's thinking. In other words, the lesson needs to be changed in ways that fit children's thinking. It is important that the teacher plans and practices mathematics lessons in a way that incorporates children's imaginations. Imagination is a primitive form of thinking and is related to creativity. The teacher makes children imagine diversely about mathematical content and exchange ideas with each other. Then, the lesson will deepen and enrich children's understanding.

This article includes lesson records which represent reform ideas: (1) Mathematics Learning with Imagination (2) Diverse View and Thinking (3) Seeing Invisible Things Numerical (4) Using Children's Imaginations Effectively. These lessons were done under the real spirit of mathematics; that is, in each sample, children have thought and imagined freely and diversely about mathematical content. As L. B. Resnick (1988) argues, mathematics should not be thought of as a well-structured subject. Thus, children would learn mathematics with much interest and their attitudes towards mathematics could become positive. Elementary mathematics lessons should be more energetic, imaginative, and creative. Fostering children's creativity is one of the most important tasks for mathematics education.

Keywords: attitude, imagination, creativity, reform, elementary mathematics

1. Introduction

Regarding elementary mathematics lessons, people attach much importance to how the teacher should effectively teach children mathematical content, which is already fixed. In addition, as the public demands higher achievement from children's learning, the teacher also requires children to get what is called "mathematical basics" such as fundamental calculation. Thus, the teacher strives very much to make children understand and acquire mathematical basics.

However, in order for children to understand and acquire the said mathematical basics, we teachers have some important things to consider. We cannot foster authentic learning for children if they do not understand mathematical content in intrinsic ways themselves. This means that the learning should be done in ways in accordance with children's own thinking. If not, they cannot fulfill their learning and understanding.

Then, what is children's intrinsic understanding and when is it fulfilled? It comes from developing mathematics learning where the teacher realizes the importance of children's internal imaginations and exchanging ideas with each other. Children have their own thinking and imaginations about mathematical content. Sometimes they have much bigger imaginations than adults, but children's imaginations include a large amount of elements that deepen mathematics learning. In other words, the teacher should create mathematics lessons considering children's intrinsic thinking and imaginations much more.

2. Rethinking and Creating Elementary Mathematics Lessons

To Improve Mathematics Education Culture

According to the report of TIMMS (1999), Japanese children's performance score of mathematics is relatively high. On the other hand, their attitudes towards mathematics are extremely negative, at the lowest level in the world. It is still a serious problem for mathematics education in Japan. The high performance may be mainly caused by learning through problem solving and a systematic curriculum. Children's negative attitudes may come from the fact that Japanese mathematics teaching is generally so fixed, for example, adopting conventional teaching processes, and setting mathematical tasks every lesson. To overcome that, we have to present new ideas and practices for reform to have children enjoy mathematics and the learning of it.

Back to the Spirit of Mathematics

Mathematics is a knowledge that has developed under the sprit of freedom, and therefore, it cannot develop under the spirit of fixation or rigidity. Even elementary mathematics is essentially the same in this sense. That is, it is important that children imagine freely and learn mathematics with rich imaginations.

For instance, look at the following interesting calculation of fractions that I saw on a TV program on mathematics several years ago:

$$\frac{2}{3}+\frac{1}{4}=\frac{2+1}{3+4}=\frac{3}{7}$$

This is a stereotypical wrong answer that can be seen in school. However, there was a man who established such a principle of calculation as a new type of mathematics. This mathematics is called Mediant Fraction. It is also known as Farey series, and can be applied to various situations. Generally, it is shown as:

$$\text{for } \frac{a}{c},\frac{b}{d} \text{ and } \frac{a}{c}<\frac{b}{d}\text{, then } \frac{a}{c}<\frac{a+b}{c+d}<\frac{b}{d}$$

This is a thought that overrules common sense. This thought had been neglected and considered a wrong answer for a long time, but was eventually approved as a good kind of mathematics. This matter shows us that it is possible for us to make new mathematics even from children’s wrong answers. It came from the fact that a man re-thought about things flexibly and grasped them in a new way.

In the same sense, the teacher can have children foster a new view and thinking even in elementary mathematics lessons. Through such mathematics learning, we can lead children to deeper understanding and expandable learning. Moreover, such mathematics learning must foster children’s interest in mathematics, and as a result, improve their achievement in mathematics.

As Resnick (1988) argues, mathematics should not be considered to be a well-structured subject or organized domain, but it should be a subject to be interpreted diversely and developed by learners. However, not only such a good idea or thought but also concrete sample practices should be presented to people, and are necessary to change conventional mathematics education and to improve mathematics lessons.

Mathematics Learning with Imagination

Imagination is a primitive and natural form of thinking, and is important in mathematics education.

The Japanese 2nd grade mathematics textbook includes the following problem regarding multiplication.

[Problem] 4 People are riding in each small car. There are 6 small cars.
Then, how many total people are riding?

When the teacher asked children what formula they could construct about this problem, they replied two kinds of answers. These were 6 x 4, and the other 4 x 6.

Counting the number of each answer, the former was 7 children; the latter was 8 children (my classroom included 15 children). As two answers were proposed, the teacher asked them which answer was right. Then, Shota (boy) referred to the formula of 6 x 4, saying "Teacher, if the formula is 6 x 4, people are not riding on small cars, but it is as if cars are riding on people." Can you imagine what Shota addressed? See the following figure (*Figure 1*).

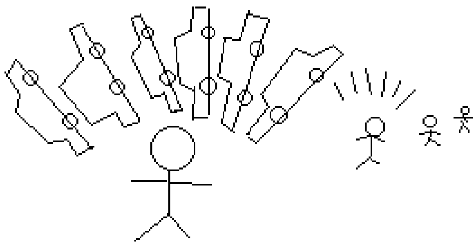

Figure 1. Shota's image about the formula of 6 x 4.

Shota had an image of the formula of 6 x 4. Furthermore, his image was shared with other children, and they were convinced that in this case, the formula of 4 x 6 is right. Therefore, the fact that some children replied 6 x 4 was not useless for the lesson; on the contrary, it became an idea to deepen the understanding of each child and was useful for the lesson.

Diverse View and Thinking

There is a problem using a dot diagram in learning multiplication. It is the following problem: What multiplication has the same value as 6 x 4?

The following diagram is attached to the problem (*Figure 2*).

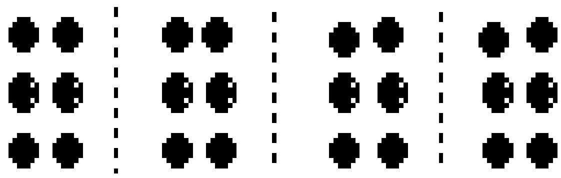

Figure 2. The diagram attached to the problem

This diagram represents 6 x 4, but the teacher had children imagine other ways of multiplication. Being asked if they could imagine other ways of multiplication while looking at this diagram, they answered two kinds of formulas, 4 x 6, and 8 x 3. Hearing their answers, the teacher asked children again, "Well, how do you see 8 x 3?" Then, the children presented five views after looking at the diagram (*Figures 3-6*).

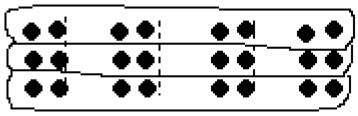

Figure 3.

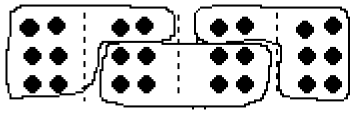

Figure 4.

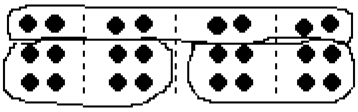

Figure 5.

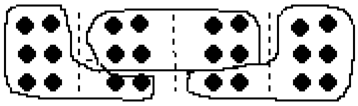

Figure 6.

The 5th view was the reverse of figure 5. These views can be said to be samples discovering invisible borderlines in a diagram. In other words, these mean the activities that children imagine diversely, in addition to expanding the images. That is, to imagine things is also to interpret things diversely, and to think about changing the form of something, or to think of it figuratively. Mathematics learning with imagination can develop and deepen children's thinking. Because all children have their own images about mathematical content, by exchanging and making use of them in mathematics lessons, the lessons could therefore deepen and broaden children's understanding. Any imagination should not be rejected as unnecessary. Even wrong answers can include a kind of value to exchange. The teacher has to try to find a value from it.

Seeing Invisible Things Numerical

By enriching children's imaginations, mathematics learning in a lesson would improve.

I will present my practice, in which I have retried mathematics lessons that Tsubota (1994) started, which find some rules about a multiplication table.

After looking at the 3's multiplication row, children began to write their thoughts in their notebooks. Then, the teacher had them express their findings. Tomomi (girl) told that the numbers in the ten's position consist of 1, 1, 1, 2, 2, 2 (*Figure 7*).

3 × 1 = 03	0 + 3 =	3	repeat
3 × 2 = 06	0 + 6 =	6	
3 × 3 = 09	0 + 9 =	9	
3 × 4 = 12	1 + 2 =	3	
3 × 5 = 15	1 + 5 =	6	
3 × 6 = 18	1 + 8 =	9	
3 × 7 = 21	2 + 1 =	3	
3 × 8 = 24	2 + 4 =	6	
3 × 9 = 27	2 + 7 =	9	

Figure 7. Tomomi's finding and idea about 3's row.

Next, the teacher asked what she thought of 3, 6, 9, and then she answered that the numbers in the ten's position were "0." In other words, though the ten's position is empty, she thought that “0” existed there. Moreover, Tomomi presented another rule she found. She said that 1 and 2 makes 3, 1 and 5 makes 6, 1 and 8 makes 9, 2 and 1 makes 3, 2 and 4 makes 6, 2 and 7 makes 9, thus the numbers of 3, 6, 9 are repeated. Adding the number in the first position to the number in the second position, it shows some rules.

Then, the teacher asked her how about 3 x 1, 3 x 2, 3 x 3, and she answered they became 0 + 3 = 3, 0 + 6 = 6, 0 + 9 = 9. Certainly, we could understand they repeated 3, 6, and 9. However, the teacher asked her a deeper question: whether they would really continue 3, 6, and 9. She nodded at hearing that. Then the teacher made further multiplications after 3 x 9 = 27 with children (*Figure 8*). Children knew how to make the 3’s row; that is, adding 3 to the former product. The teacher wrote 3 x 10 = 30, 3 x 11 = 33, and 3 x 12 = 36 on the blackboard. Looking at them, children understood the products also continue 3, 6, and 9.

$$3 \times 10 = 30 \rightarrow 3 + 0 = 3$$
$$3 \times 11 = 33 \rightarrow 3 + 3 = 6$$
$$3 \times 12 = 36 \rightarrow 3 + 6 = 9$$

Figure 8. Tomomi’s developed idea regarding of the 3’s row

However, after the teacher wrote 3 x 13 = 39, and 3 + 9 = 12, the teacher told the children it did not make the number 3. Here, the rule found by Tomomi seemed to have collapsed. Nevertheless, regarding this problem, Sachiko (girl) said, “Teacher, 1 and 2 makes 3! “ Her idea is as follows:

3 x 13 = 39   3 + 9 = 12 (not 3)   but   1 + 2 = 3

That is, though 3 + 9 = 12 is not suited to Tomomi’s rule, when we consider the product of 12 consists of 1 + 2, we can get 3 also.

Using Children’s Imaginations Effectively

The 2nd grade teaching materials regarding shapes include the unit “Let’s make dice”. It is the learning material children face in the 3rd term and becomes a basis of the learning of cubes.

First, the teacher had children make sure of how many edges a die has, and then passed out graph paper to them. Next, the teacher had them draw developed figures of a cube that were likely to become dice on their own. After a while, some children presented their developed figures (*Figures 8-9*).

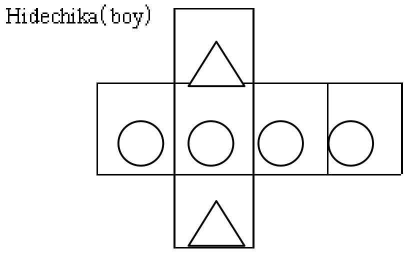

Figure 8.

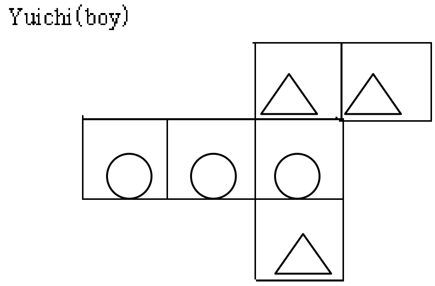

Figure 9.

Regarding these figures, the teacher asked children whether they had noticed anything. Then, Hidechika (boy) answered, "There are 4 faces horizontally, and 2 faces above and below (see *Figure 8*)." In addition to that, he addressed, "Only Yuichi's figure is different." Certainly, Yuichi's is different, having 3 faces horizontally and 3 faces above and below. That is, his idea means that there are 3 faces with a circle symbol and 3 faces with a triangle symbol (see *Figure 9*).

From this sentence, the current content deals partly with a plan that I have not practiced actually, that is, a supposed lesson plan. Towards Hidechika's response, I should have asked children, "But Yuichi's figure must have something the same or common to the others, must it not?" because when any die is made, there must be some commonness among developed figures. I was so attracted by this problem that it was regrettable that I did not assure children sufficient time to think about that. Looking at these figures, we can find a rule when we pay attention to the borderlines on both Yuichi's figure and the others. Paying attention to the borderlines between one face and another face, we can recognize that each figure has 3 vertical borderlines and 2 horizontal borderlines (*Figure 10*).

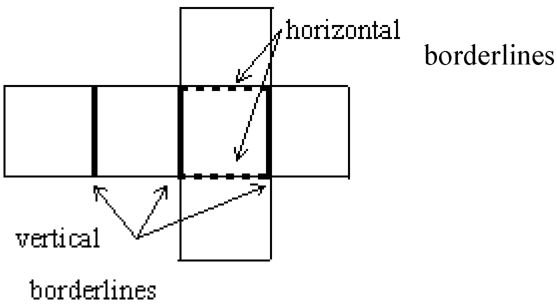

Figure 10. A rule found in developed figures of a cube

I should have waited until children had found out this rule. On the contrary, I told them, "Somehow, each figure seems to have 3 vertical borderlines and 2 horizontal borderlines." However, this learning will be useful for them to prepare for future learning, for example, the famous Euler's Theorem: $v + f - e = 2$ (v= vertices, f= faces, e= edges)

3. Conclusions

In the practice of “Let’s make dice,” I should have made children do trial and error through much more thinking. The teacher should not impatiently teach an important thing to children. It is important that the teacher waits for children’s thoughts to grow little by little. While waiting for them, their imaginations will expand and their ideas will be formed.

Mathematics lessons incorporating children’s rich imaginations means learning in which the teacher makes great importance of children’s imaginations. Furthermore, when the teachers, as well as children, face a teaching material with interest, discovering new problems and searching for the solutions together, such teaching and learning will be accompanied by much thrill and fun. That is, it would give us a genuine feeling of being alive during lessons. That very feeling fosters children’s interest in mathematics and desire to learn it. As a result, it will improve children’s attitudes towards mathematics as well as their performance.

Elementary mathematics lessons should be more energetic, imaginative, and creative. This is because imagination is closely related to creativity, and to foster children’s creativity is one of the most important tasks for developing mathematics education. Through fostering children’s creativity, not only mathematics education but also our society can develop towards a better future.

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