Developing Elementary Mathematics Lessons and Curriculum Standards to Foster Children's Mathematical Thinking and Expression Abilities

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Introduction

Generally speaking, people emphasize that elementary school children should be taught just basic mathematic fundamentals and knowledge, such as basic calculation or understanding the names and the properties of simple diagrams. In such lessons, teachers teach mathematics lessons in a way that children do not think deeply about mathematical matters, that is, children just learn the procedures of calculation, the properties of diagrams, etc. I am sure that children should learn basic and fundamental skills and knowledge, however, such lessons do not promote interest and appreciation of mathematical learning. I think that children do need to think for themselves and act for themselves through mathematics lessons in addition to mathematic fundamentals.

Then, how can we foster children's abilities to think and express themselves mathematically? I will present my fundamental ideas and an actual record of lessons.

How We Can Foster Children's Mathematical Thinking and Expression Abilities

Add Advanced Learning in Mathematics Curriculum or Develop the Curriculum

Curriculum standards show explicitly minimum essentials that children have to learn in school. Japanese textbooks are written based on the curriculum standards and it has no extension because of limitation. It is a teacher's responsibility to lead his/her lesson to develop children's thinking beyond the restrictions of the textbook. Furthermore, we need to plan advanced learning in our mathematics curriculum within limited teaching hours. This means that we should intersperse the curriculum standards with expanded content. Children's mathematical thinking and expression abilities are fostered through such advanced learning (see *Figure 1*).

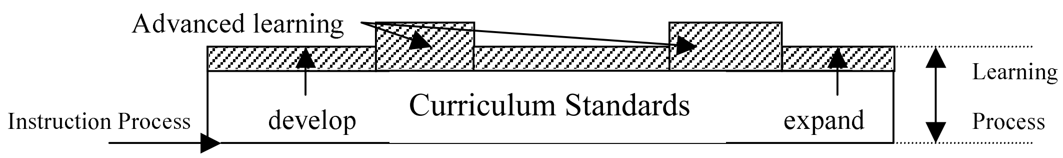

Figure 1. Model of the relation between curriculum standards and advanced learning

Within a teaching unit: put advanced learning in curriculum standards daily.

Changing from "Knowledge Pouring-type Lessons" to "Knowledge Creating-type Lessons"

Next, we teachers need to change our views of elementary mathematics lessons. We should try to change from traditional "Knowledge Pouring-type Lessons" to "Knowledge Creating-type Lessons." Knowledge pouring-type lesson means a traditional and conventional style where teachers give children ready-made mathematics knowledge and concepts in a one-sided way. On the contrary, knowledge creating-type lesson means that children create mathematics knowledge and concepts themselves. This change is needed for children as well as for teachers. It is important that children feel that learning mathematics is to create new mathematics thinking, while talking about mathematical matters together.

The Way of Thinking about "Knowledge Creating-type Lessons"

The necessary condition to approve "Knowledge Creating-type Lessons" is the thought that elementary mathematics lessons consist of children's constructive activities (the view of constructive learning). Constructive learning is done through children's collaborative and interactive learning about each thought. Therefore, teachers do not explicitly teach children their knowledge and skill but children grasp a problem themselves, pursue, and solve the problem. Thus, they create new knowledge themselves through the learning of problem solving. (But, I do not agree to what is called "New-New Math," overemphasizing constructive learning without basic drills.)

Until now it has been said that such learning is only for advanced children; however, this method could be used to deepen mathematical understanding in low-level children also. The situation where the class includes various types of children all thinking together in mathematics lessons makes it possible to practice knowledge creating-type lessons. That is, in normal classrooms of mixed abilities, we can make use of children's thoughts, developing their learning and creating new knowledge. Because of this we do not need to group children specially.

The Role of Teacher is to Draw Out Children's Thoughts

What is the way to be able to make use of low-level children's thoughts as I stated in the above chapter? It means that children's simple and natural questions and thoughts could be important elements of mathematics lessons. That is, children's expressions, words, attitudes, and responses could be elements that are integrated into mathematics lessons. Sometimes, children's murmuring and mistakes can be important cues. The teacher needs the ability to draw out children's expressions and to teach children how to express their thoughts. The teacher has to draw out children's thoughts, listen to them, and let children discuss them together, and by doing so expanding their thoughts. In other words, a teacher would act as a chair presiding over a lesson or as the coordinator of it.

When in a Lesson Children's Mathematical Thinking and Expression Abilities Should be Advanced

Although we might say we can do it in every lesson stage, it is first clear that we can encourage children when they ask questions. Children discover new problems when facing various mathematical teaching materials that the teacher presents in a lesson. When discovering new problems, they have to think about it in many ways and express their thoughts.

Next, their abilities can be developed at the stage where they pursue their problem cooperatively. Then, they have to think out and express their thoughts and solutions. Moreover, to analyze the expression presented also can become a matter to think about together. Therefore, it is important that we teachers lead children to discover other new problems after they solve the previous problem. That is, knowledge creating-type lessons show continuous problem solving from problem to problem. Such lessons make it possible to draw out and encourage children's thinking and expression. The point is that we let children think about and discuss mathematical matters, which can be a method to foster children's mathematical thinking and expression abilities (*Figure 2*).

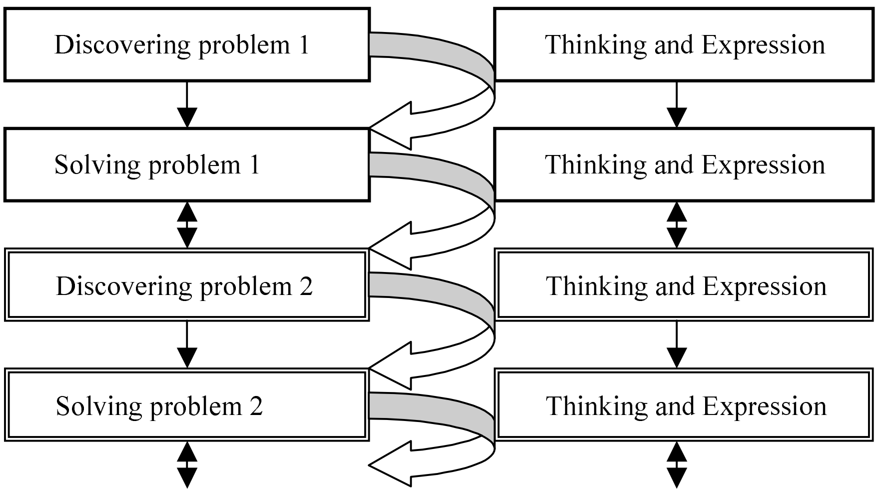

Figure 2. Phases of continuous problem solving lessons: Mathematical thinking and expression appear in discovering and solving a problem, and continuing in this way.

Sample of "Knowledge Creating-type Lesson"

Objective: Discovering the formula of the areas of quadrangle figures through measurement of a kite diagram.

Planning advanced learning in the unit of "5th grade: The Area of Quadrangle and Triangle."

The Unit Plan (17 hours)

Area of Parallelogram--------------------3 hours

Area of Triangle--------------------------3 hours

Area of Various Quadrangles------------5 hours (includes the lesson here)

Review-------------------------------------2 hours

Further learning on how to measure the area of other diagrams---4 hours

Sample of practice.

I presented to the children the following kite diagram (see *Figure 3*) and made them think about how to measure the area.

The children showed various ways of measuring it as follows (Solving problem):

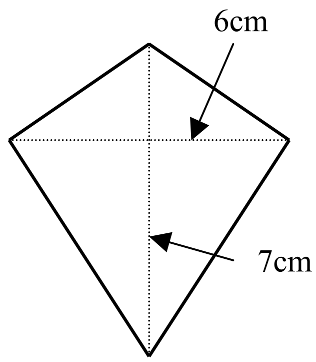

Figure 3. The beginning problem.

[Solution 1]

Child A calculated the area of two triangles. (see *Figure 4*)

$7 \times 3 / 2 \times 2 = 21(cm^2)$

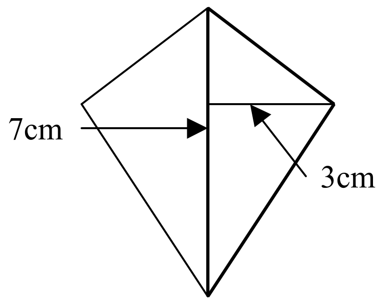

Figure 4. Solution 1.

[Solution 2]

Child B inserted two triangles into a rectangle and calculated the area of the rectangle (see *Figure 5*). $7 \times 3 = 21(cm^2)$

[Solution 3]

Child C made a large rectangle seen in *Figure 6* surrounding the kite diagram. Then he calculated the area of the large rectangle and divided by two.

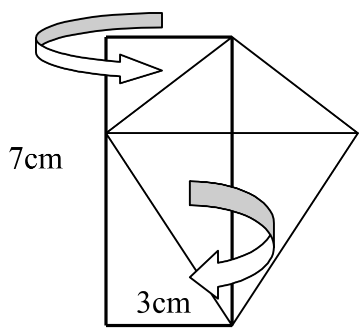

Figure 5. Solution2. (insert two triangles)

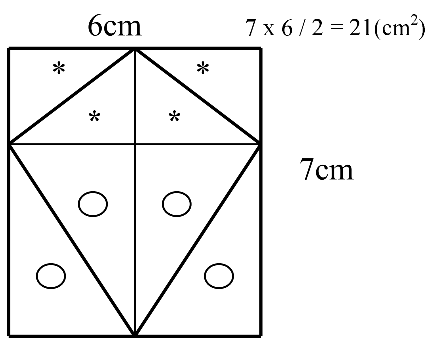

Figure 6. Solution 3. (measure using the double area)

[Solution 4]

Child D cut the kite diagram into two. He reversed one triangle, attached it to the other one and made a parallelogram (see *Figure 7*).

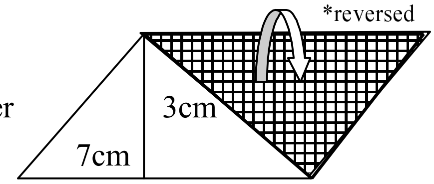

Figure 7. Solution 4.

7 x 3 = 21 (cm^2)

Next, I let children think of the formula to measure the kite diagram.

We found that we had to use "diagonal lines" which had not been used in measuring the diagrams at all. We came to notice that their areas could be measured with the formula, "diagonal line x diagonal line / 2" (one diagonal line length multiplied by another diagonal line length divided by 2). Therefore, the result was "7 x 6 / 2."

Though they thought it was over, I asked them about the properties of this kite diagram formula. Looking at the diagram, the children found that the two diagonal lines were perpendicular (see *Figure 8*). That is, the children had foreseen that they might measure the area of the diagram with perpendicular diagonal lines using the formula, "diagonal line x diagonal line / 2."

But this lesson was not over yet. Next, the lesson developed towards a new problem whether we could find other diagrams that we might apply this formula to (Discovering problem).

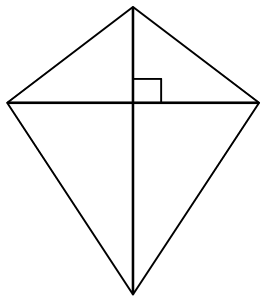
Figure 8. Diagonal lines are perpendicular.

Our concern changed to the problem of whether there are any diagrams with two perpendicular diagonal lines like the kite diagram. Then, children answered, "It is a rhombus" and found that the area of a square could also be calculated by the formula (Solving problem). The children found that new problem can be solved by this formula and our problem solving was not over. Another teacher continued my practice and our problems.

Broadening and Deepening Practice

Reading the above lesson record, Tanaka (2003), developed our problems with his 5th graders and furthered it (the below is a summary):

> From the knowledge that if the diagrams have two perpendicular diagonal lines we could use the former formula, children foresaw that they might also apply the formula to the following diagram (see *figure 9*). (Discovering problem and solving problem) This is a trapezoid (quadrangle with unequal

sides). When we transform this diagram, it becomes a right triangle (see *Figure 10*). Right triangle is a kind of quadrangle with two perpendicular diagonal lines and, therefore, the same formula can be applied. Tanaka tried to transform the quadrangles further, and he and his students found that they could use the formula in the case of the following diagram (see *figure 11*). (Solving problem) Generally, this diagram is called a “chevron.”

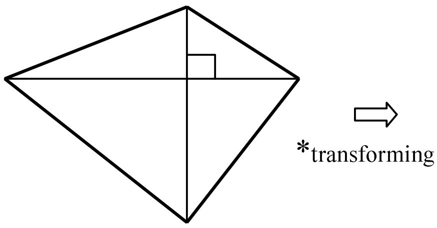

Figure 9. Applied diagram.

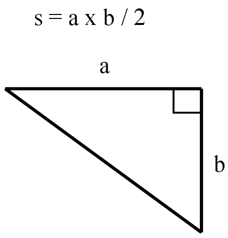

Figure 10. Right triangle.

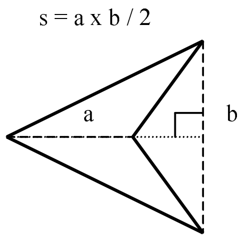

Figure 11. Chevron.

Conclusion

Recently, it has been said loudly that we should reform our mathematics education. But frankly speaking, we cannot say it has been very fruitful. Various proposals have been made about how to teach mathematics lessons, however, I have not heard mathematics classroom in school has been actually reformed. The reason why lesson reform has been less successful may be that teachers have not considered much of children’s thoughts and feelings in mathematics lessons until now. Therefore, we need to accept and consider children’s thinking and expression much more. In other words, we need to reform our lessons into continuous problem solving lessons that foster children’s mathematical thinking and expression abilities. Through continuing to practice lessons that make children think and express themselves mathematically, children acquire basic skills and knowledge. It is not too much to say that basic mathematic fundamentals can be acquired through advanced learning and developing curriculum standards. Moreover, teachers also need to develop new teaching materials in order to create lessons fostering children’s mathematical thinking and expression abilities. It is important that teachers study the subject of mathematics themselves and teach children mathematics creatively.

Reference

Tanaka, H. (2003). Taikakusen x Taikakusen / 2 de motome rareru zukei. [Diagrams that can be measured by the formula of the diagonal line x diagonal line / 2.]. *Elementary Mathematics Lesson Study*. 27, 20. (In Japanese)